Microsoft Research Cambridge

Probabilistic programming the future of Machine Learning?

John Winn

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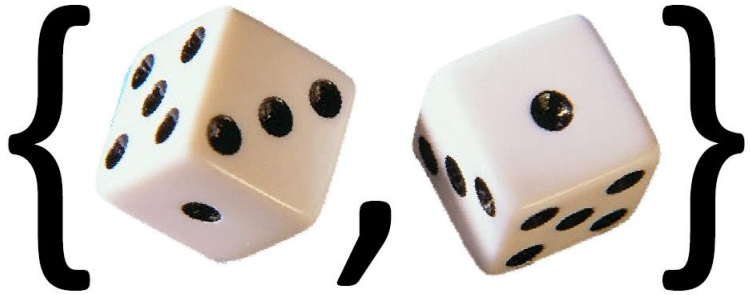
Outline

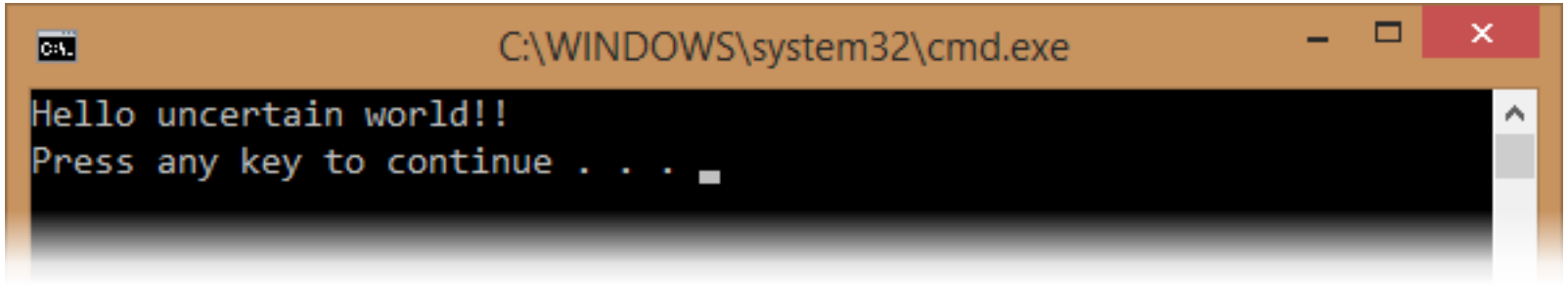
- What is probabilistic programming?
- Why is it important for machine learning?
- Example: interpreting user behaviour
- Example: parsing text
- Getting probabilistic programs to run quickly



What is probabilistic programming?

Hello uncertain world (C#)

```
// Create two strings
string a = 50%:“Hello uncertain” 50%:“Hello”
string b = 50%:“world” 50%:“uncertain world”
// Format strings together into a new string
string c = a + “ ” + b + “!!”;
// Write it out
Console.WriteLine(c);
```



In probabilistic C# (Csoft)

```
// Create two strings
string a = Random(Strings.Uniform());
string b = Random(Strings.Uniform());
// Format strings together into a new string
string c = a + " " + b + "!!";
```

Uniform over strings

Random variables

- Normal variables have a fixed single value:

```
int length=6,
bool visible=true.
```

- Random variables have uncertain value specified by a probability distribution:

```
int length = Random(Uniform(0,10));
bool visible = Random(Bernoulli(0.8));
```

- `Random` means ‘is distributed as’.

Observed data (constraints)

- We can define observations on random variables:

```
Observe(visible==true)
Observe(length==4)
Observe(length>0)
Observe(i==j)
```

- `Observe(b)` means ‘we observe b to be true’.

Inference

- `Infer` gives the posterior distribution of one or more random variables.
- Example:

```
int i = Random(Uniform(1,10));
bool b = (i*i>50);
var bdist = Infer(b);//Bernoulli(0.3)
```

- Output of `Infer` is always *deterministic* even when input is *random*.

Example: linear regression (x to y)

```
// Unknown line parameters
double a = Random(Gaussian(0,1000));
double b = Random(Gaussian(0,1000));
// Loop over points
for (int i=0; i < x.Length; i++) {
  // Equation of line
  double clean_y = a * x[i] + b;
  // To match the data, we must add some noise.
  Observe(y[i] == Random(Gaussian(clean_y,1)));
}
var adist = Infer(a); // Learn slope
var bdist = Infer(b); // Learn intercept
```

Semantics: sampling interpretation

Imagine running the program (very) many times:

- `Random(d)` *samples* from the distribution `d`.
- `Observe(b)` *discards* the run if `b` is false.
- `Infer(x)` *stores* the value of `x`.
  - If enough `x`'s have been stored, returns their distribution.
  - Otherwise stops the run and starts a new run.

Example: linear regression (x to y)

```
// Unknown line parameters
double a = Random(Gaussian(0,1000));
double b = Random(Gaussian(0,1000));
// Loop over points
for (int i=0; i < x.Length; i++) {
  // Equation of line
  double clean_y = a * x[i] + b;
  // To match the data, we must add some noise.
  Observe(y[i] == Random(Gaussian(clean_y,1)));
}
var adist = Infer(a); // Learn slope
var bdist = Infer(b); // Learn intercept
```

Running probabilistic programs

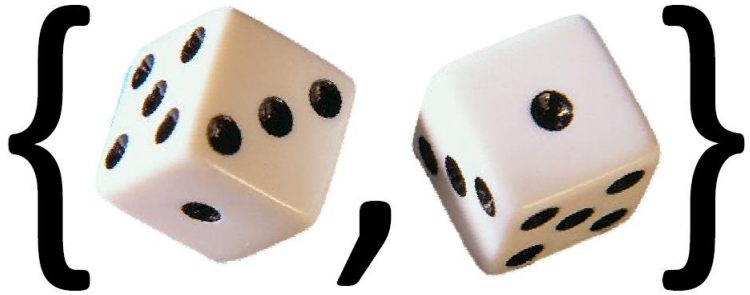
Probabilistic programs can be:

- Interpreted on-the-fly e.g. Church, Dimple
- Compiled to generate algorithm code e.g. Infer.NET, Stan, OpenBUGS

Our Infer.NET compiler:

- Runs deterministic message passing
  e.g. Expectation Propagation, Variational Message Passing
- Supports a wide range of probabilistic programs
- Scales up to very large data sets (GB, TB or more)
- Used in hundreds of applications (internal and external)
- Now 10 years old!

http://research.microsoft.com/infernet



Why is probabilistic programming important?

Case study: How Good Are You at Halo?

Xbox Live

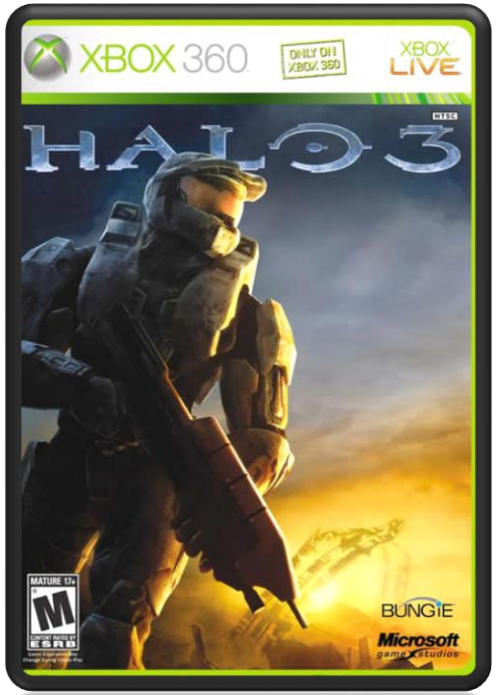
- 12 million players
- 2 million matches per day
- 2 billion hours of gameplay

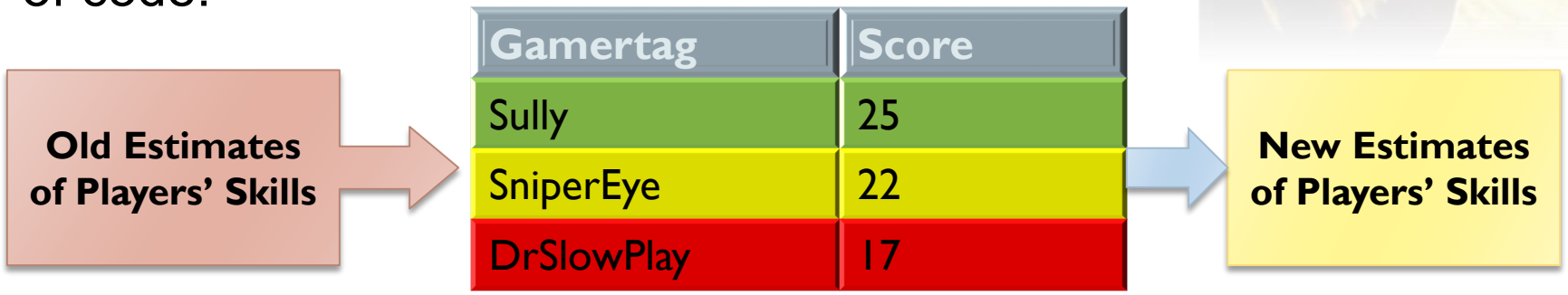
The Challenge

- Tracking how good each player is to match players of similar skill.

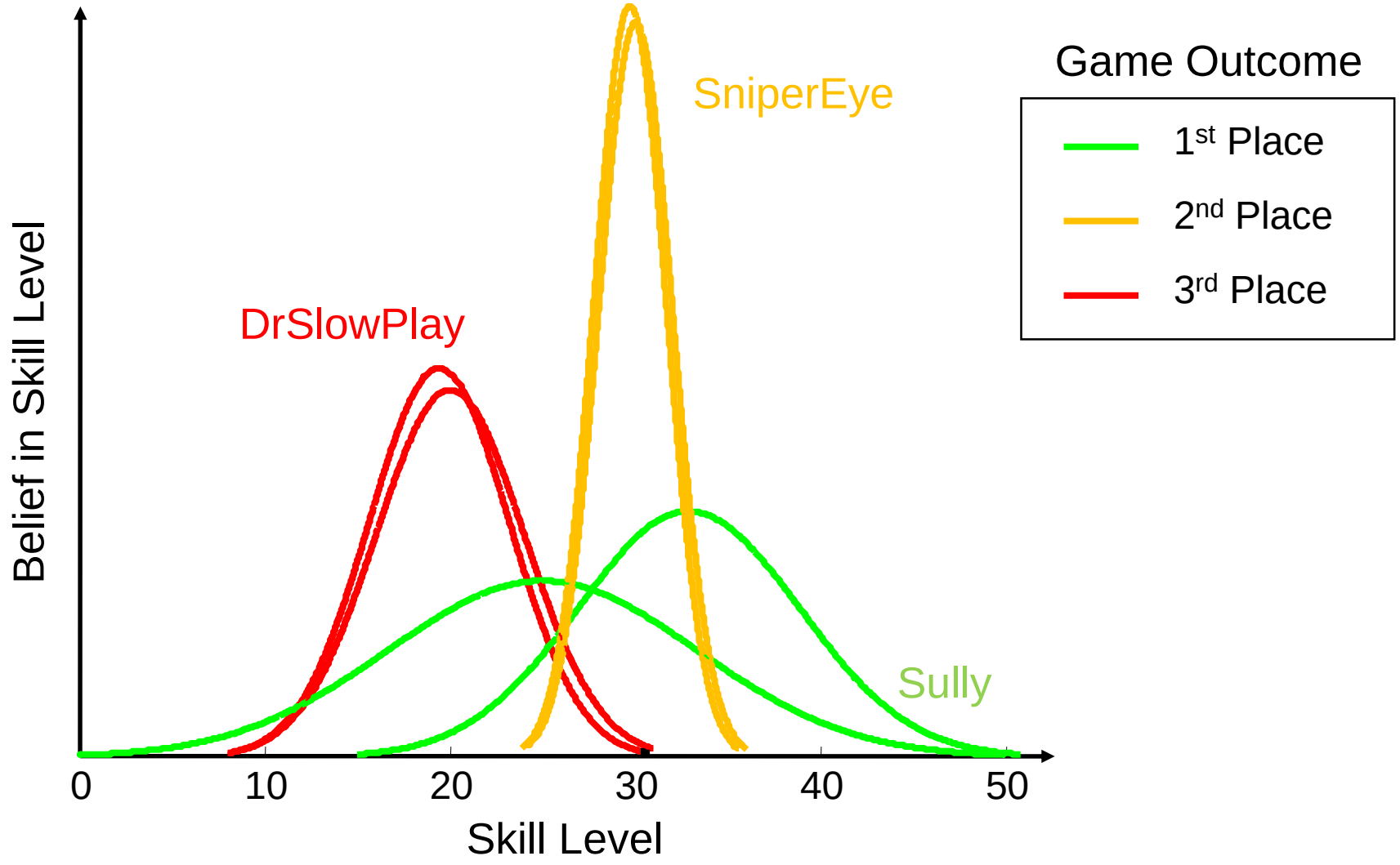
TrueSkill™

- Months of work, algorithm was 100s of lines of code.





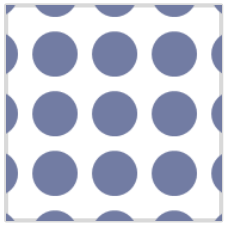
Inferring Skills



TrueSkill™ in Probabilistic C#

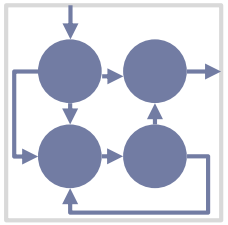
```
double[] skill=new double[nPlayers];
double[] performance=new double[nPlayers];
for (int j = 0; j<nPlayers; j++) {
  skill[j] = Random(Gaussian(means[j],vars[j]));
  double noise = Random(Gaussian(0, beta));
  performance[j] = skill[j] + noise;
  if (j>0) Observe(performance[j-1] > performance[j]);
}
return Infer(skill);
```

Machine learning is becoming…



More **common**

Ever more places where machine learning plays a key role.



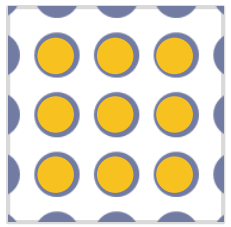
More **complex**

Used to address increasingly challenging problems.

More **large scale**

Larger datasets, coming from more diverse sources.

…more common

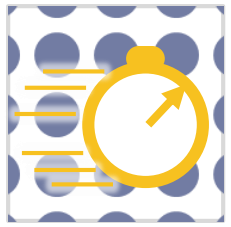


Probabilistic programming requires (much) less expertise.

» more people can use machine learning

Probabilistic programs are more transparent.

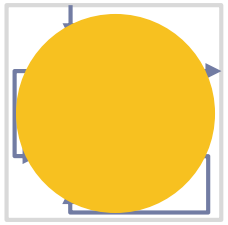
» experts can provide support more easily



Probabilistic programs are short.

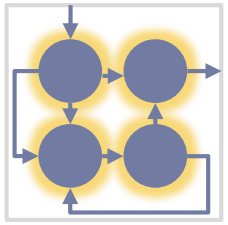
» quicker to write and experiment (play) with

…more complex



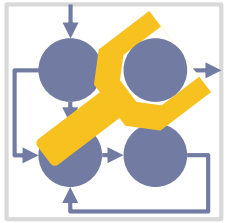
Probabilistic programming hides complexity of Bayesian methods.

» helps break the ‘complexity barrier’



Re-usable inference engines.

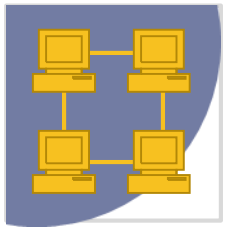
» thoroughly tested, fewer bugs



Probabilistic development tools.

» help to manage complexity

…more large scale



Probabilistic programs say *what* problem to solve, not *how* to solve it.

» multiple possible execution back ends

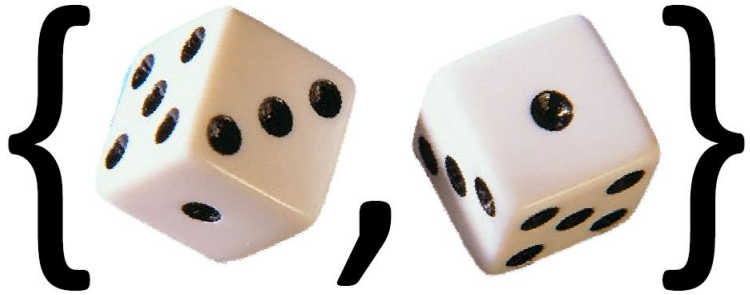
(Mobile, CPU, GPU, Cluster, Cloud)

Probabilistic programs allow auto-parallelisation.

» Exploits the structure of the program

Probabilistic programs allow online inference.

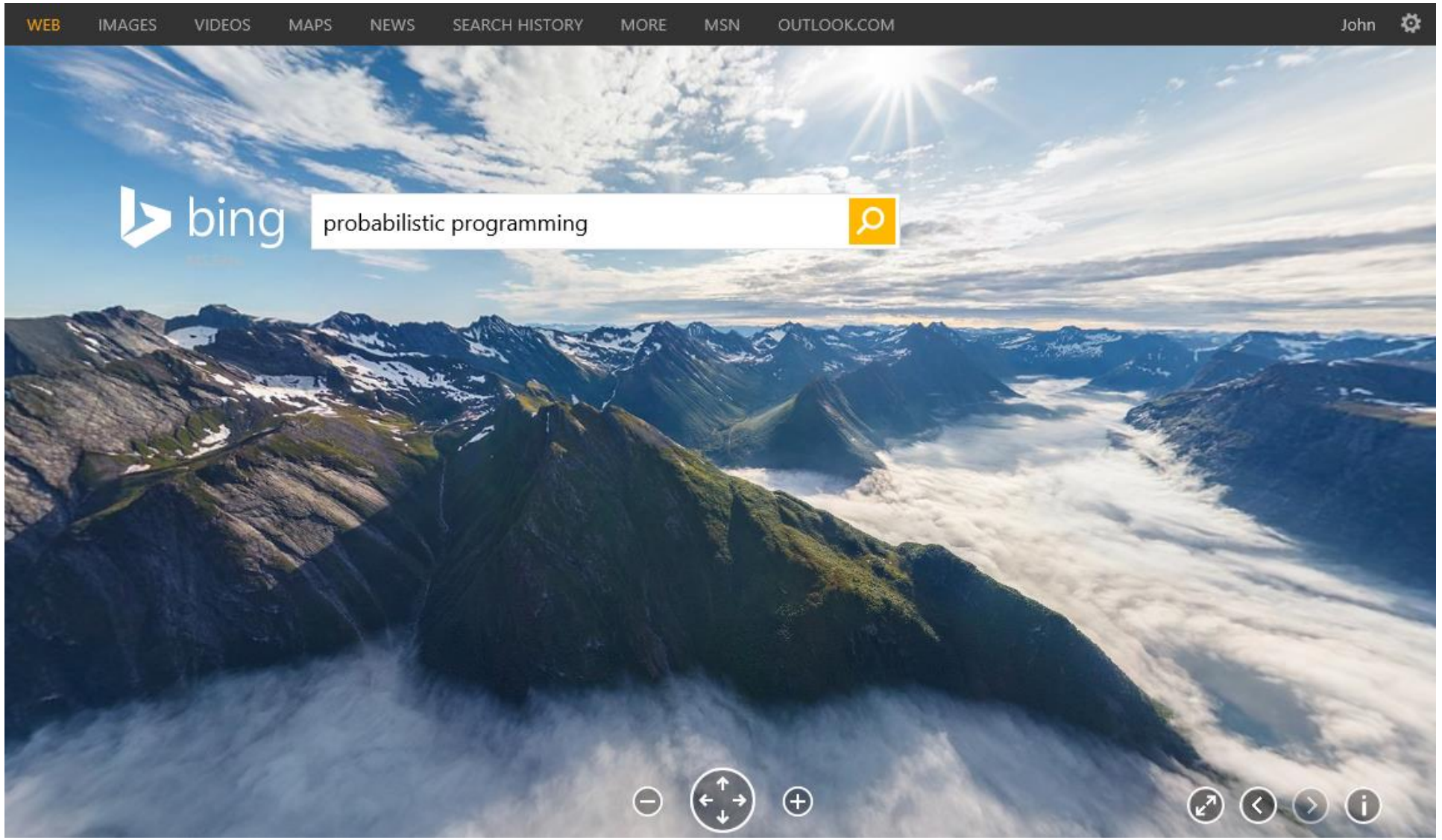
» Batch & online versions from one program



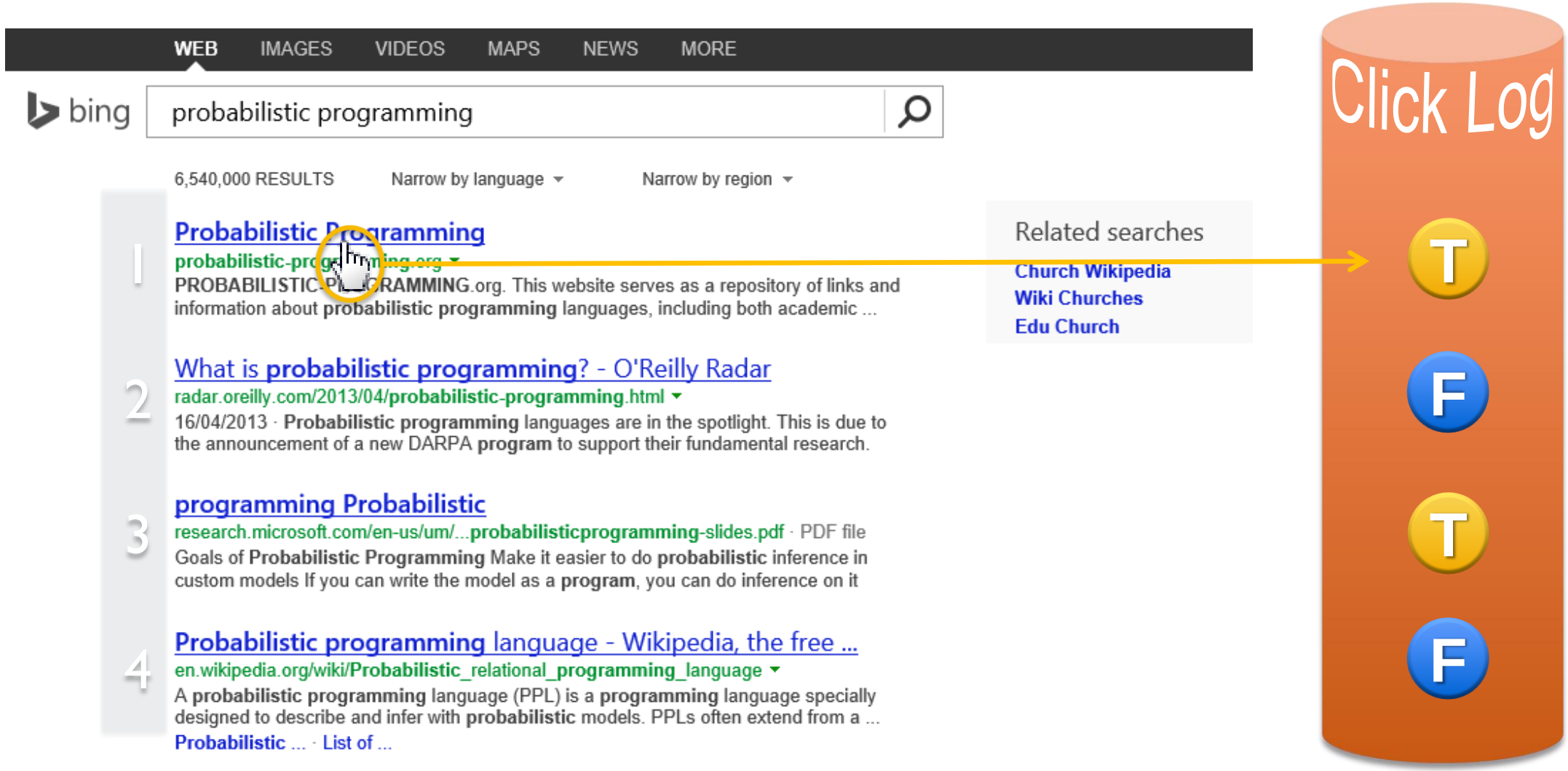
Probabilistic programming for interpreting user behaviour

With Tom Minka, John Guiver

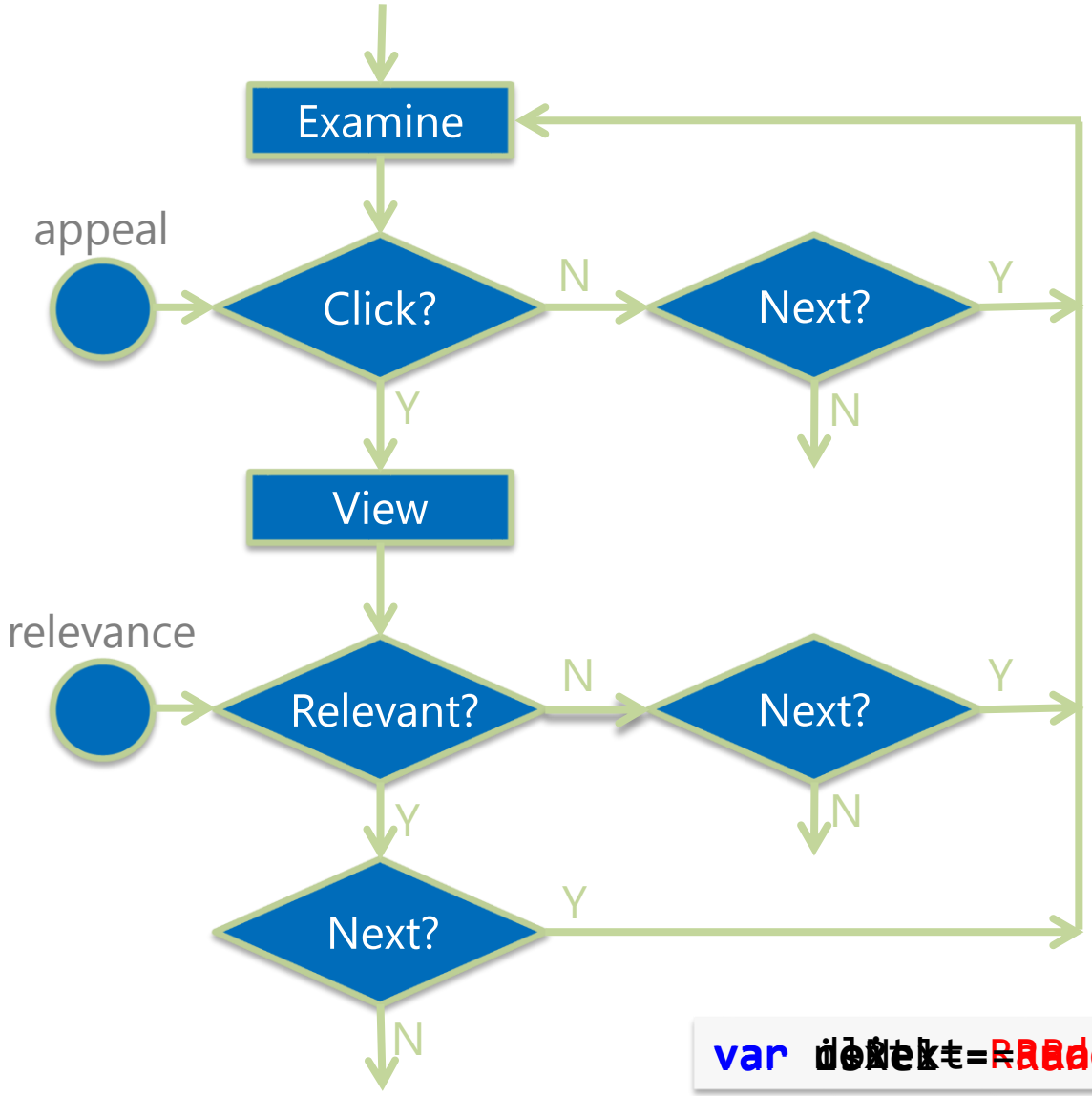
Example : Search Log Analysis



The Click Log



Programming a user




```
var
```

Programming a user (code snippets)

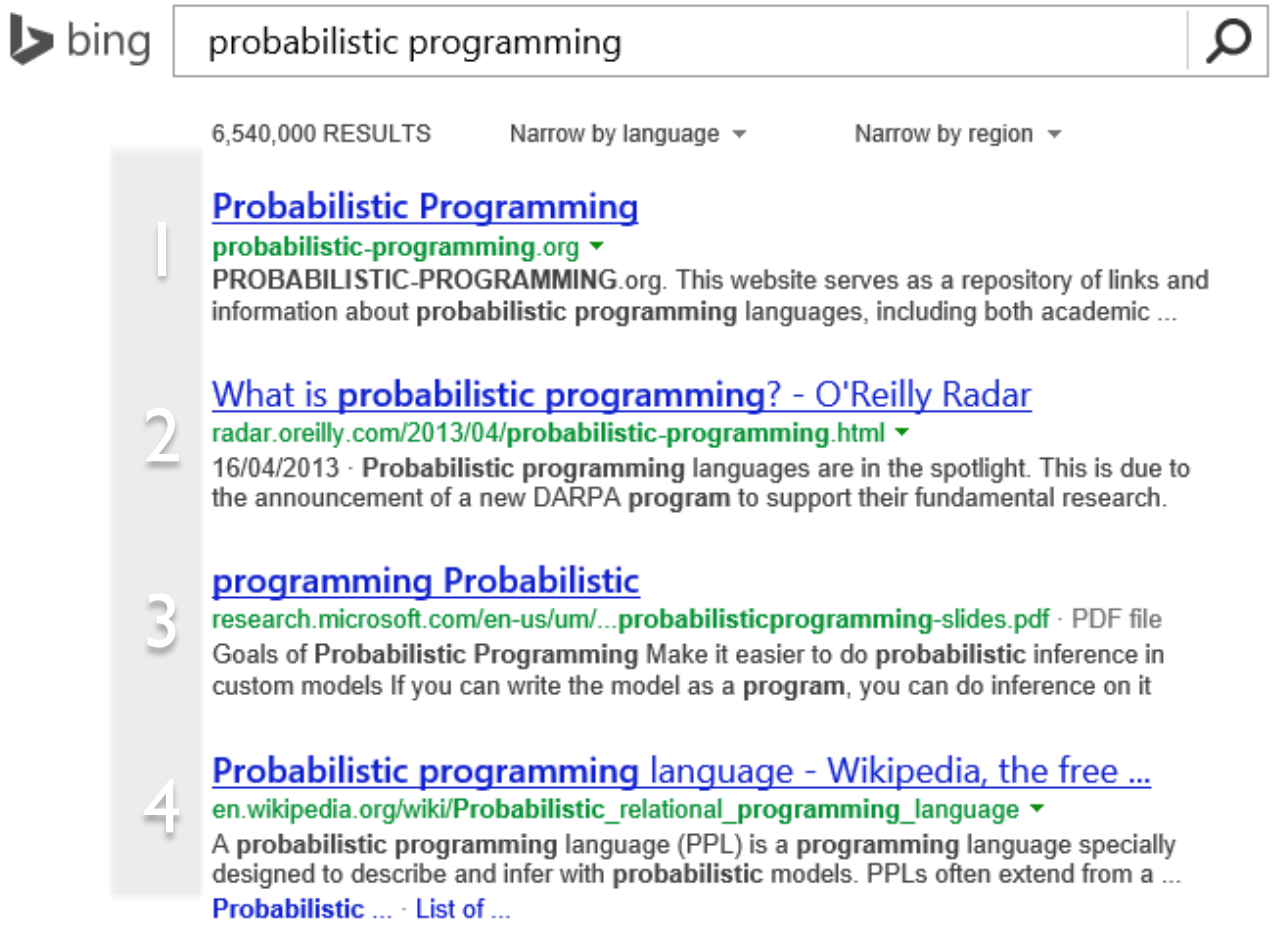
```
appeal[d] = Random(Beta(1,1));     // Unknown appeal
relevance[d] = Random(Beta(1,1)); // Unknown relevance
```

```
// For each user/document
// Should user examine the next search result?
examine[d] = examine[d - 1] &
  ((((!click[d - 1]) & nextIfNotClick) |
     (click[d - 1]  & nextIfClick));
```

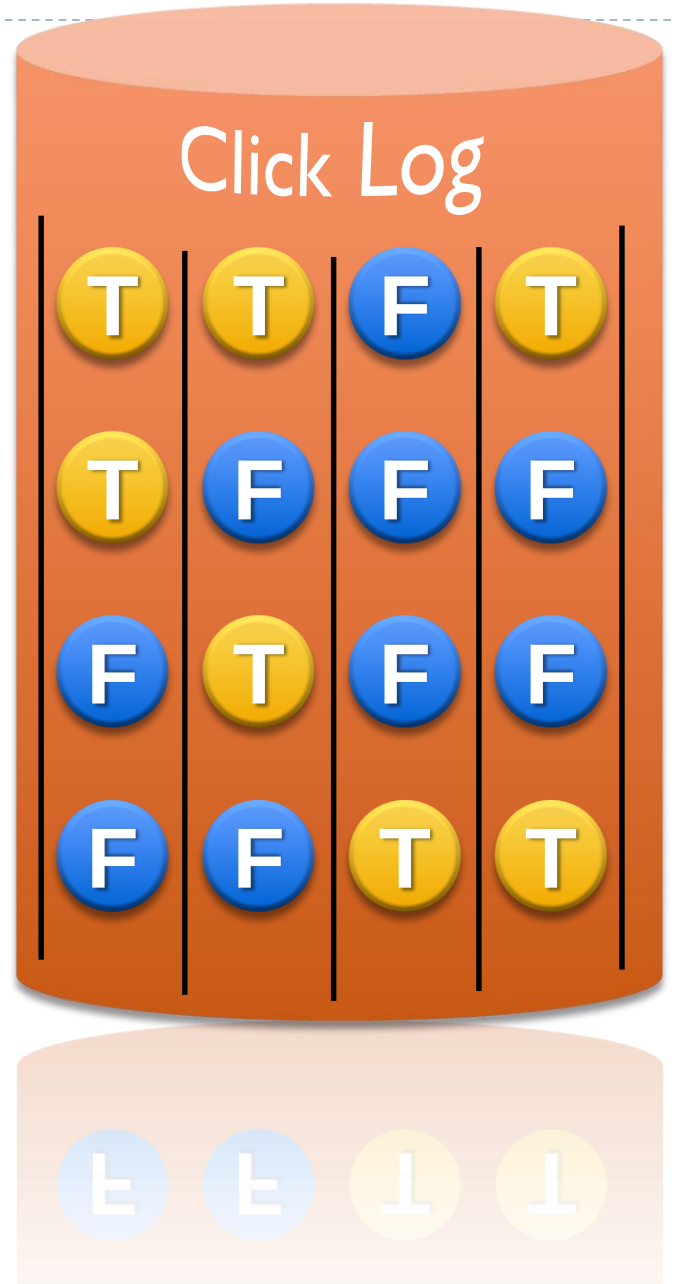
```
// User clicks if they examined result and it appealed to them
click[d] = examine[d] & Random(Bernoulli(appeal[d]));
```

```
// User finds relevant page if they clicked and was relevant
isRelevant[d] = click[d] & Random(Bernoulli(relevance[d]));
```

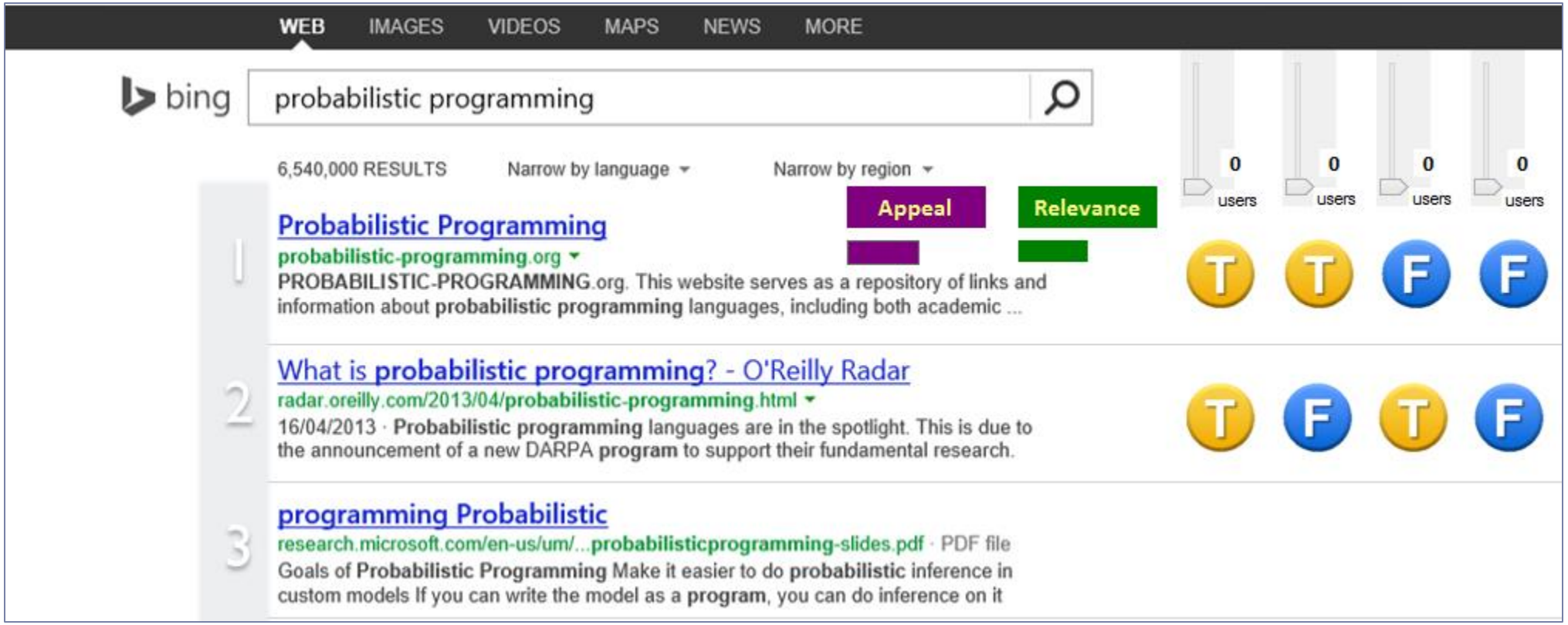
Attaching data to the program

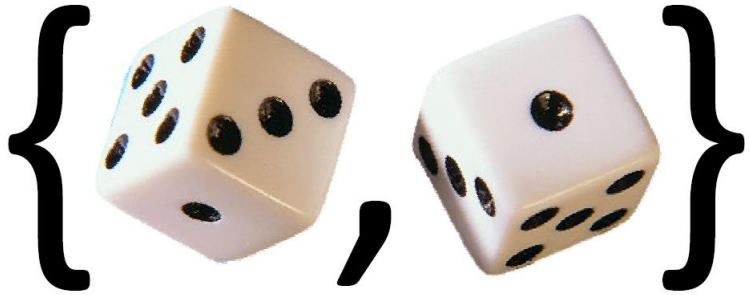



```
for (int d = 0; d < nRanks; d++)
    Observe(click[d]==user.clicks[d]);
```



Clickthrough Demo





Probabilistic programming for parsing text

With Tom Minka, Boris Yangel

Example 1: learning a name

```
// Pick a name
string name = Strings.Capitalized();
// Format it into a text string
string text = String.Format("My name is {0}.", name);


// Observe result
Observe(text, "My name is John.");
// Infer name
var namedist = Infer(name);
```

←A random capitalized string

←Returns 100%:"John"

Example 2: learning a template

```
// Pick a name
string name = Strings.Capitalized();
// Pick a template
string template = Strings.Any() + Chars.Nonword() + "{0}"
                        + Chars.Nonword() + Strings.Any();
// Format name into a string using the template
string text = String.Format(template , name);
// Observe result
Observe(text, "My name is John.");
// Infer template
var tempdist = Infer(template);
```

A random non-word character

←Returns 100%:"My name is {0}."

Example 2: learning a template

```
// Pick a name
string name = Strings.Capitalized();
// Pick a template
string template = Strings.Any() + Chars.Nonword() + "{0}"
                        + Chars.Nonword() + Strings.Any();
// Format name into a string using the template
string text = String.Format(template , name);
// Observe result
Observe(text, "Hello! My name is John.");
// Infer template
var tempdist = Infer(template);
```

50%:"Hello! My name is {0}."
50%:"Hello! {0} name is John."

Example 3: shared template

```
// Pick two names
string name1 = Strings.Capitalized();
string name2 = Strings.Capitalized();
// Pick a template
string template = Strings.Any() + Chars.Nonword() + "{0}"
                        + Chars.Nonword() + Strings.Any();
// Format names into strings using the template
string text1 = String.Format(template , name1);
string text2 = String.Format(template , name2);
// Observe texts
Observe(text1, "Hello! My name is John.");
Observe(text2, "Hello! My name is Andy.");
```

Now template is 100%:"Hello! My name is {0}."

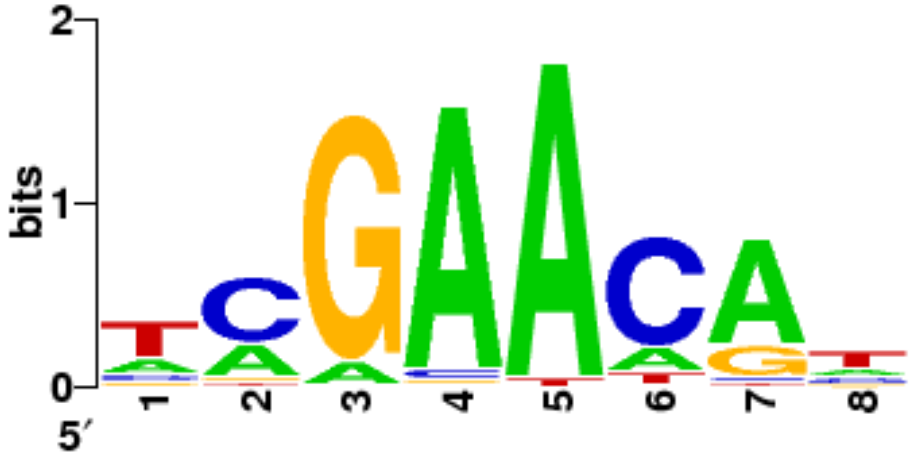
Example 4: non-string values

```
// Pick a name and date
string name = Strings.Capitalized();
DateTime date = Dates.Any();
// Convert date to string
string dateFormat = Strings.OneOf("d MMM, yyyy", "d/M/YY");
string dateString = date.ToString(dateFormat);
// Pick a template
string template = "{0}" + Strings.StartEndN
                  "{1}" + Strings.StartNon
// Format names into strings using the template
string text = String.Format(template , name, dateStr);
// Observe texts
Observe(text, "Fred was born on 6 May, 1963.");
```

←A random DateTime

name = "Fred"
date = 6/5/1963
dateFormat = "d MMM, yyyy"
dateString = "6 May, 1963"
template = "{0} was born on {1}."

Motif finding



TAGAAAGT
TCGAACAC
ACAAACGT
TAGCACAA

Motif finding: data (synthetic)

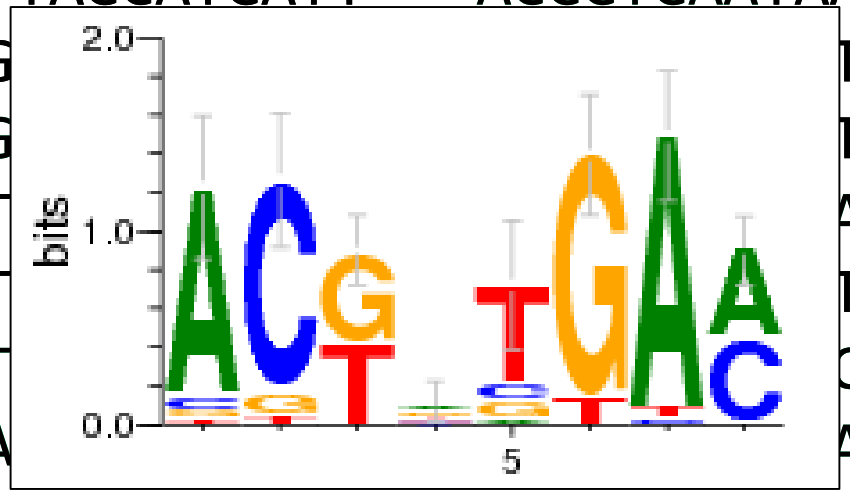
```
CGTGACGGTTACCGCTTCCTATTTG    AATTTGCTCCTAGCAACTGTGAAGA
CATAGAGGCGCGATGAACATGAGAC    AGTGCCGCGGACTATGTCTGTTACA
GGGAATAACTATGACTTACTTGCGG    CGACTGCAACGAACTGTGACATTAA
CGTCGGTGAAAATATTAGCATGATT    ACGGTGAATAAGGTCGATAATGGTT
CTTAACTTCGTCGTGAAACTTTGAC    TTGTGCGCAATTCGCTAACTCTGAA
CCCTAAACAACGAAGAAACCGTTTA    CACGGTGAAATTCGTCGCTCTCAGT
AGGACTGTGACCCATCGTTGACGCT    GACCACTGGGAAGGAAACAGCCCGC
CCCCGGACGTTGACTATCTAATGCA    CACGTGCCCATCCGATGAACTGATC
GAGTGCTCTAGCACTATGAATACGT    GCGACTTTGTCTCTATATGATTATG
CTGGGATCTAGACTATTACAGACGA    GTCACCTGTGACGCCGTGCGGTTTA
GGCACGGGGAGTAAGTCTGAAATGC    CAAGATGCTGGTTCAACGTCGAAAA
CTTAGGCCACGGTTAAGCAACAGGC    GGGAAACAGTACGAGTAGGACTCAA
CACAACGGTGCCTAACGTATCTTAA    TACTGTAACGATGACTGCCGGCCGC
TATCTACGATGAATCACCCAAATAT    GCACATTACGATGAACAATGCCATT
GGTCAGGAATTATGACATGCATTGT    CGAAACCGCGACGTTACGTTGACCC
```

Motif finding: probabilistic program

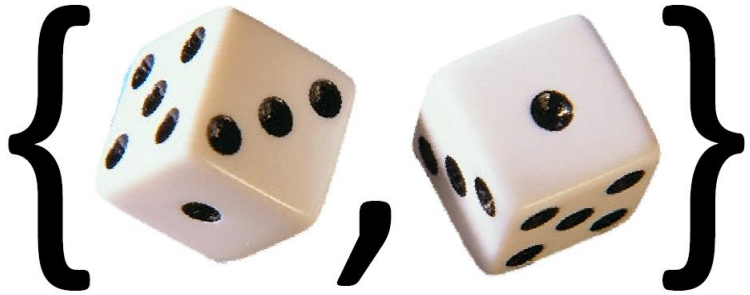
```
// Generate motif i.e. base probabilities
Vector[] motifProbs = new Vector[motifLen];
for (int i=0; i < motifLen; i++) motifProbs[i] = Random(dirichletPrior);
// Loop over sequences
for (int j=0; j < seqs.Length; j++) {
  // Generate motif instance
  for (int i=0; i < motifLen; i++) motif[i] = (char)Random(motifProbs[i]);
  // Choose motif position
  int motifPosition = Random(Discrete(0,seqs[j].Length-motifLen));
  // Generate left and right background strings
  string left = Strings.OfLength(motifPosition, backgroundDist);
  int rightLength = seqs[j].Length - motifPosition - motifLen;
  string right = Strings.OfLength(rightLength, backgroundDist);
  // Observe result
  Observe(seqs[j] == left + motif + right);
}
```

Motif finding: results

CGTGACGGTTACCGCTTCCTATTTG AATTTGCTCCTAGCAACTGTGAAGA
CATAGAGGCGCGATGAACATGAGAC AGTGCCGCGGACTATGTCTGTTACA
GGGAATAACTATGACTTACTTGCGG CGACTGCAACGAACTGTGACATTAA
CGTCGGTGAAAATATTAGCATGATT ACGGTGAATAAGGTCGATAATGGTT
CTTAACTTCGTCGTG TTCGCTAACTCTGAA
CCCTAAACAACGAAG TTCGTCGCTCTCAGT
AGGACTGTGACCCAT AAGGAAACAGCCCGC
CCCCGGACGTTGACT TCCGATGAACTGATC
GAGTGCTCTAGCACT CTCTATATGATTATG
CTGGGATCTAGACTA ACGCCGTGCGGTTTA
GGCACGGGGAGTAAGTCTGAAATGC CAAGATGCTGGTTCAACGTCGAAAA
CTTAGGCCACGGTTAAGCAACAGGC GGGAAACAGTACGAGTAGGACTCAA
CACAACGGTGCCTAACGTATCTTAA TACTGTAACGATGACTGCCGGCCGC
TATCTACGATGAATCACCCAAATAT GCACATTACGATGAACAATGCCATT
GGTCAGGAATTATGACATGCATTGT CGAAACCGCGACGTTACGTTGACCC



PREDICTION OVERLAP GROUND TRUTH



Making probabilistic programs run faster (or at all!)

With Nicholas Heess, Danny Tarlow and Ali Eslami

Two probabilistic programming systems

Probabilistic C# & Infer.NET

```
bool[] ProbitRegression(VectorGaussian weightsPrior,
                        Vector[] features)
{
  Vector weights = Random(weightsPrior);
  bool[] labels = new bool[features.Length];
  for (int i = 0; i < features.Length; i++)
  {
    double score = Vector.InnerProduct(weights, features[i]);
    double noise = Random(Gaussian(0,0.1));
    labels[i] = (score + noise) > 0;
  }
  return labels;
}
```

Probit Regression Classifier

Church

```
(define (DP alpha proc)
  (let ((sticks (mem (lambda x (beta 1.0 alpha))))
        (atoms (mem (lambda x (proc)))))
    (lambda () (atoms (pick-a-stick sticks 1)))))

(define (pick-a-stick sticks J)
  (if (< (random) (sticks J))
      J
      (pick-a-stick sticks (+ J 1))))

(define (DPmem alpha proc)
  (let ((dps (mem (lambda args
                    (DP alpha
                        (lambda () (apply proc args))
                        )))))
    (lambda argsin ((apply dps argsin))) ))
```

Dirichlet Process

How inference works

Probabilistic C# & Infer.NET

- Deterministic message passing for large set of built-in factors e.g. EP, VMP.
- Detailed understanding of the program structure and functions used.
- Messages can be passed in either direction.

+ Very fast inference/large data

– Less flexible modelling language

Church

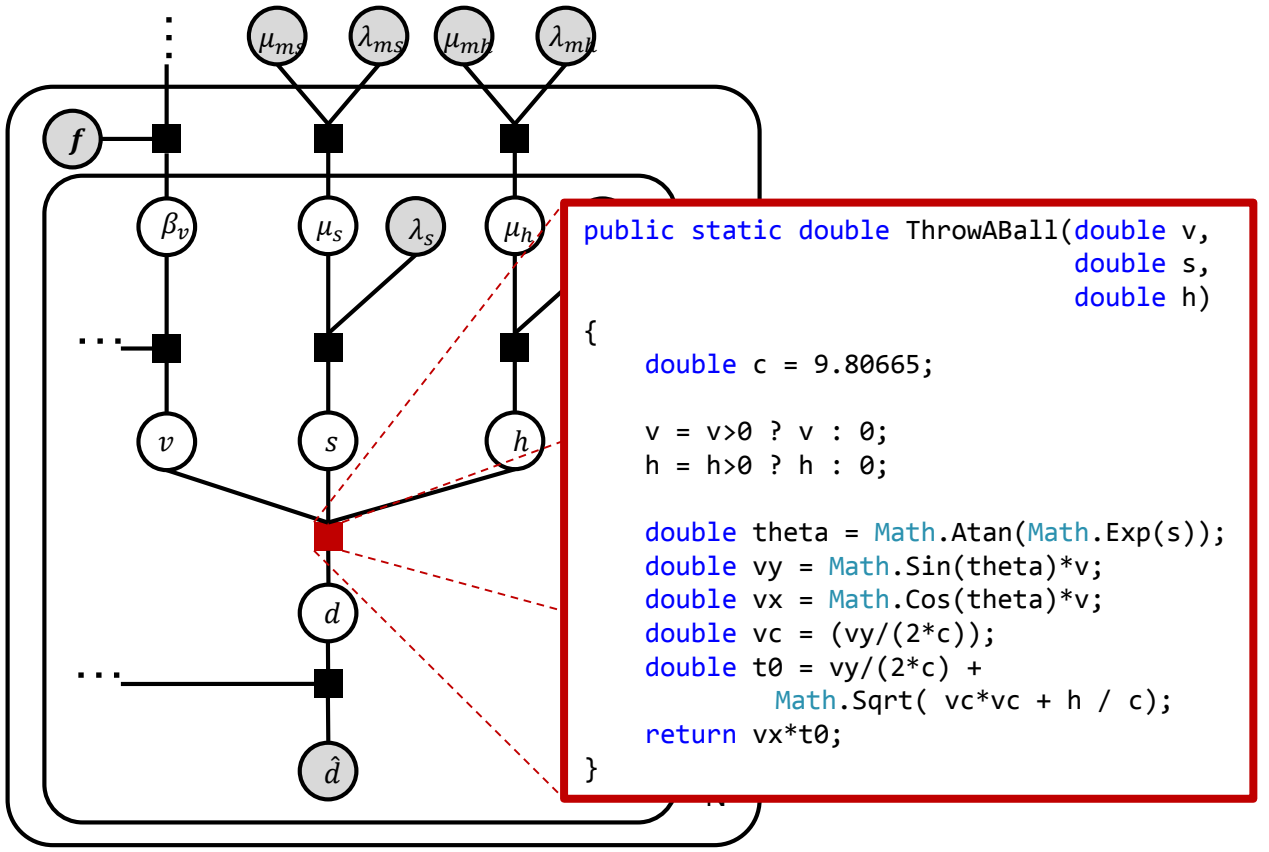
- Various sampling approaches e.g. MH, MCMC
- Program treated as a ‘black box’ (more recent work uses dependencies)
- Program only run forwards.

+ Very flexible modelling language

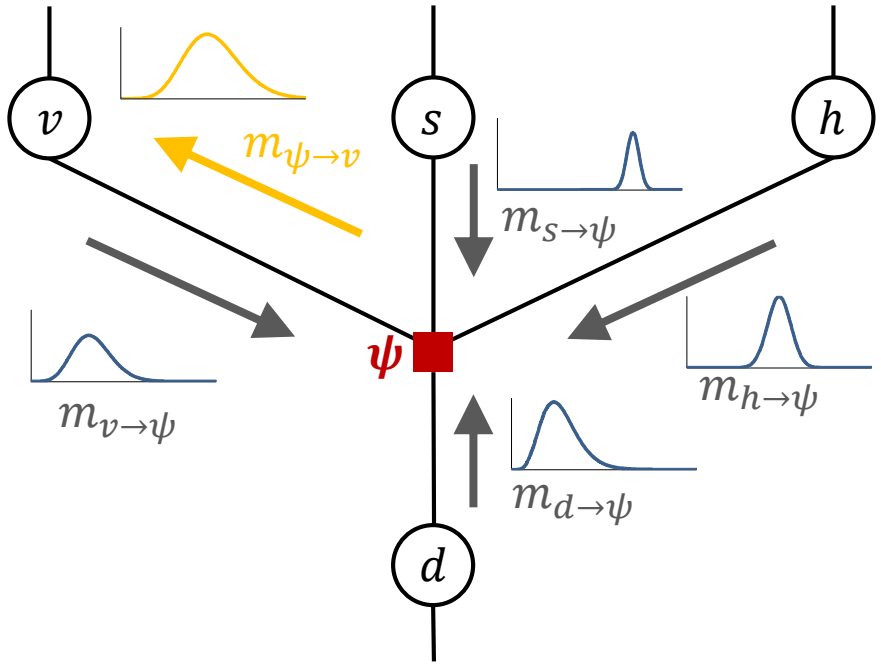
– Slower inference/smaller data

Best of both worlds?

1. Define an *arbitrary* factor using its forward program.
2. Use sampling methods to (very slowly) compute the messages coming from the factor.
3. Train a fast regression model to predict outward messages from input messages.
   e.g. random forest/neural net
4. Use regression model to do fast, scalable inference in the entire graph.



Expectation Propagation



Variable-to-factor message

$$m_{\psi i}(x_i) = \frac{\text{proj}\left[\int \psi(x_{out} \mid x_{in}) \left(\prod_{i' \in Scope(\psi)} m_{i'\psi}(x_{i'})\right) dx_{\psi - i}\right]}{m_{i\psi}(x_i)}$$

Hard to compute – Infer.NET mainly uses hand-crafted numerical approximations.

Instead – *learn* to compute these messages.

Training data via sampling

Using some proposal distribution q over input variables, compute variable-to-factor message:

$$\int \psi(x_{out} \mid x_{in}) \left(\prod_{i' \in Scope(\psi)} m_{i'\psi}(x_{i'}) \right) dx_\psi = \mathbb{E}_r \left[\frac{\prod_{i' \in Scope(\psi)} m_{i'\psi}(x_{i'})}{q(x_{in})} \right]$$

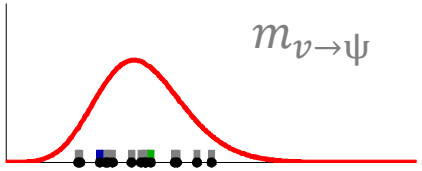
where $r(x) = q(x_{in})\psi(x_{out} \mid x_{in})$. In the simplest case: $q(x_{in}) = \prod_{i' \in in} m_{i'\psi}(x_{i'})$.

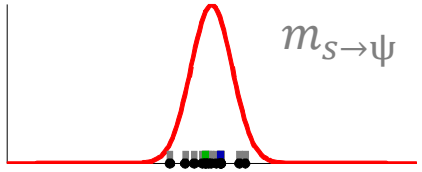
[See also ABC-EP, S. Barthelme and N. Chopin.]

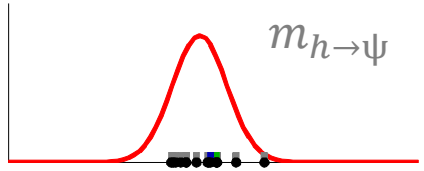
Need also to specify a distribution over input messages or use messages encountered as the inference runs.

Train neural net/random forest to predict output from input.

① Sample from incoming messages







$v_k \sim Gam(\alpha_v, \beta_v)$

$s_k \sim N(\mu_s, \lambda_s^{-1})$

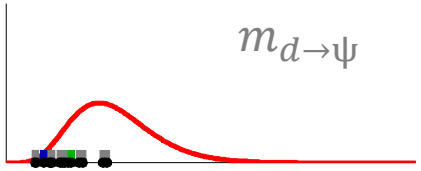
$h_k \sim N(\mu_h, \lambda_h^{-1})$

ψ

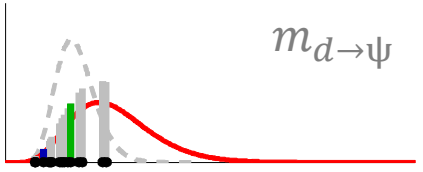
② Sample factor function

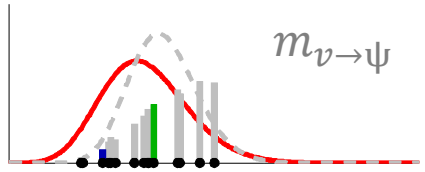
$d_k = f(v_k, s_k, h_k)$

$\sim \psi(\cdot\,|v_k, s_k, h_k)$



③ Compute IS weights $w_k = Gam(d_k \mid \alpha_d, \beta_d)$ e.g.

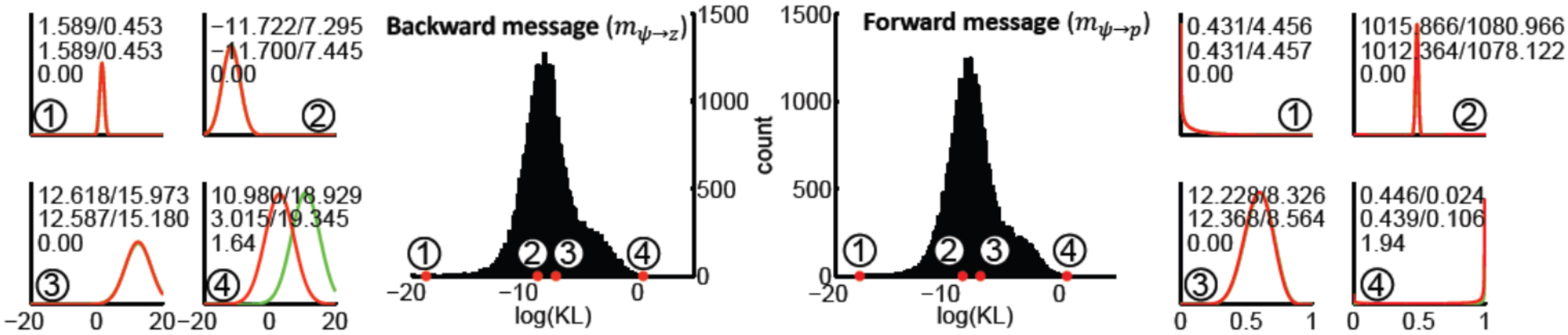




④ Moments of weighted samples, e.g. for v

$$\hat{\mu} = \frac{\sum_k w_k v_k}{\sum_k w_k} \qquad \hat{\sigma}^2 = \frac{\sum_k w_k (v_k - \hat{\mu})}{\sum_k w_k}$$

Logistic Factor $f(x)=1/(1+\exp(-z))$

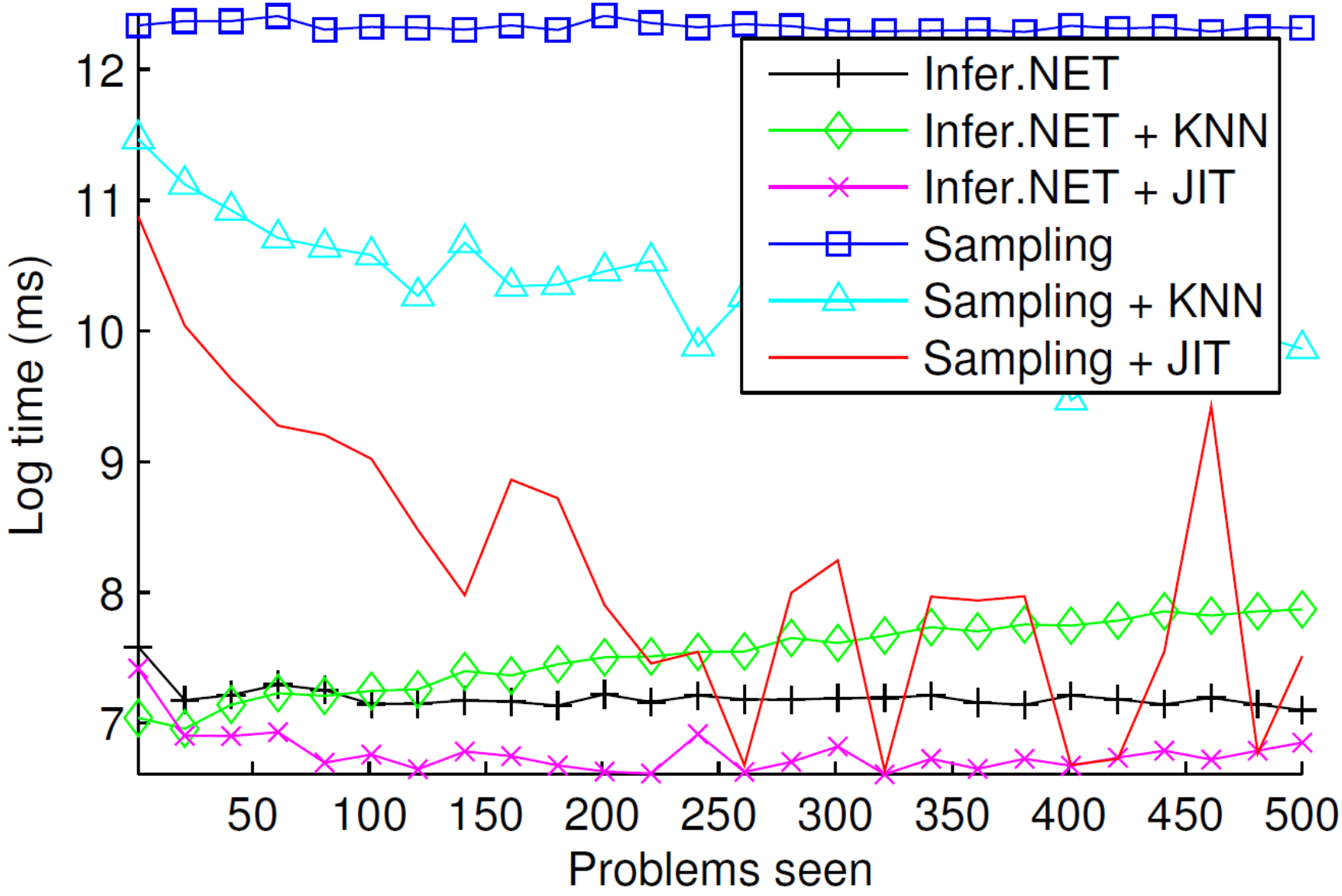


Logistic regression experiments (artificial data)

Dataset	D	D_{rel}	N	Prior var.	EP Test acc.	NN Test acc.	$KL(EP\|\|NN)$
1	10	4	500	.1	0.7750	0.7745	1.9770
1	10	4	500	1	0.7750	0.7750	1.3247
2	50	10	500	.1	0.7635	0.7635	0.4649
2	50	10	500	1	0.7660	0.7660	0.5641
4	50	40	500	.1	0.8825	0.8820	0.6022
5	50	40	100	1	0.8065	0.8050	0.0847

Test accuracy on UCI ionosphere dataset: 0.8839 (default factor) vs. 0.8800 (learned NN factor).

Speed up over sampling

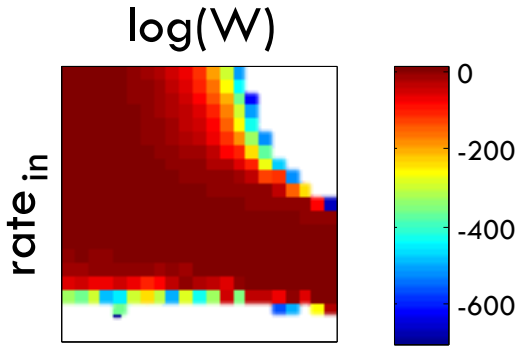


Compound Gamma

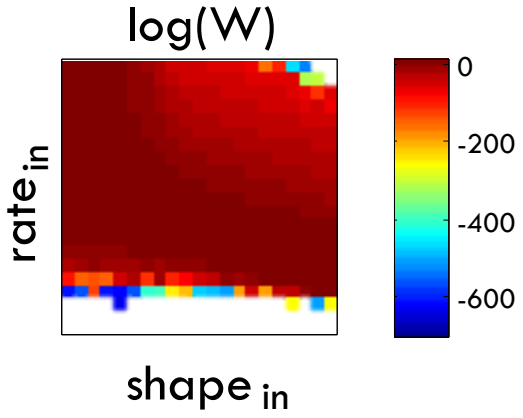
$$f() \sim \int Gam(\cdot|\alpha_B, \beta) Gam(\beta|\alpha_A, \beta_A) \mathrm{d}\beta$$

IS comparison

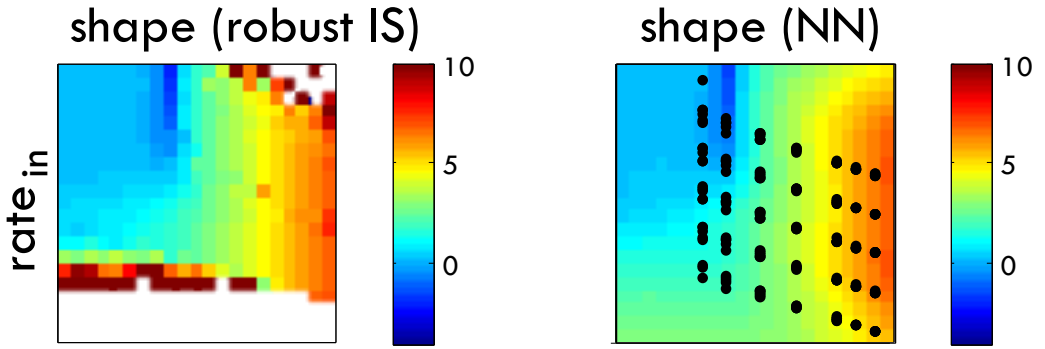
Naïve q

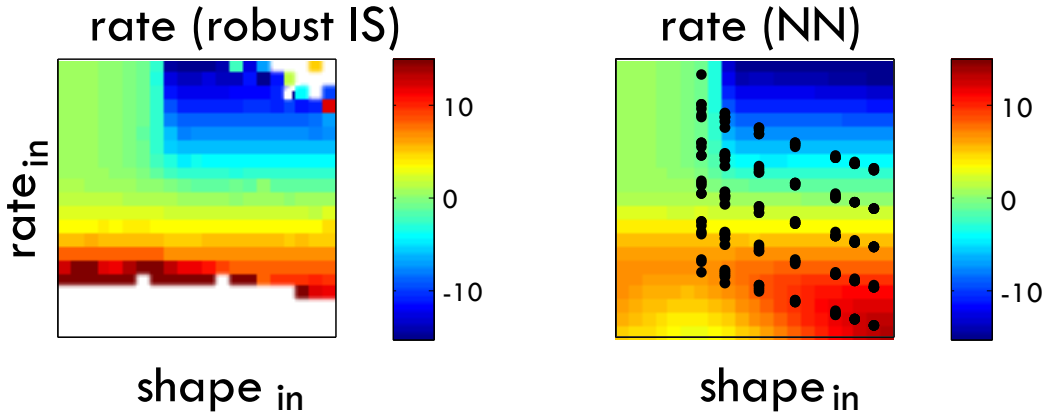


Robust q

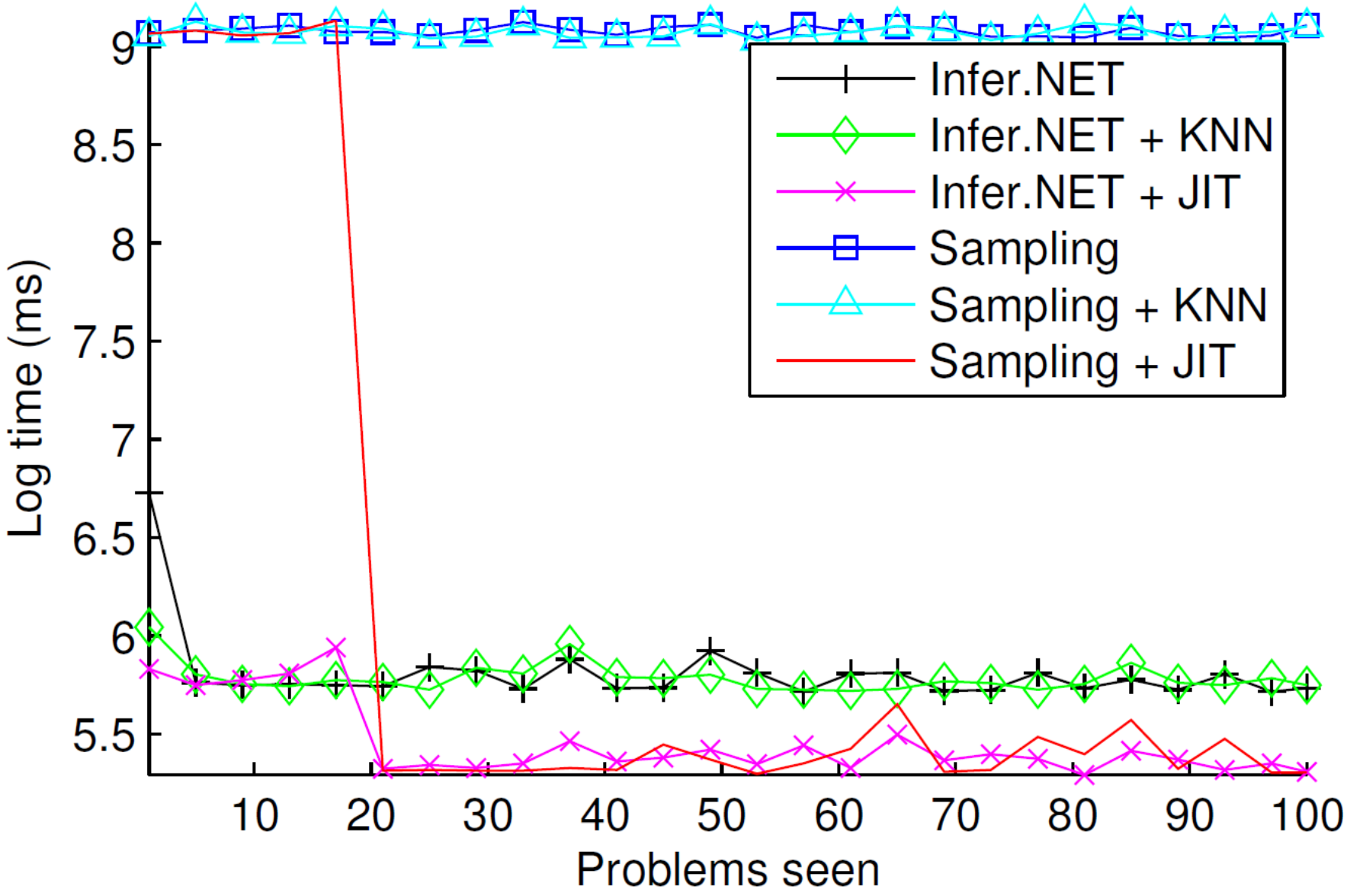


Message surfaces for robust q

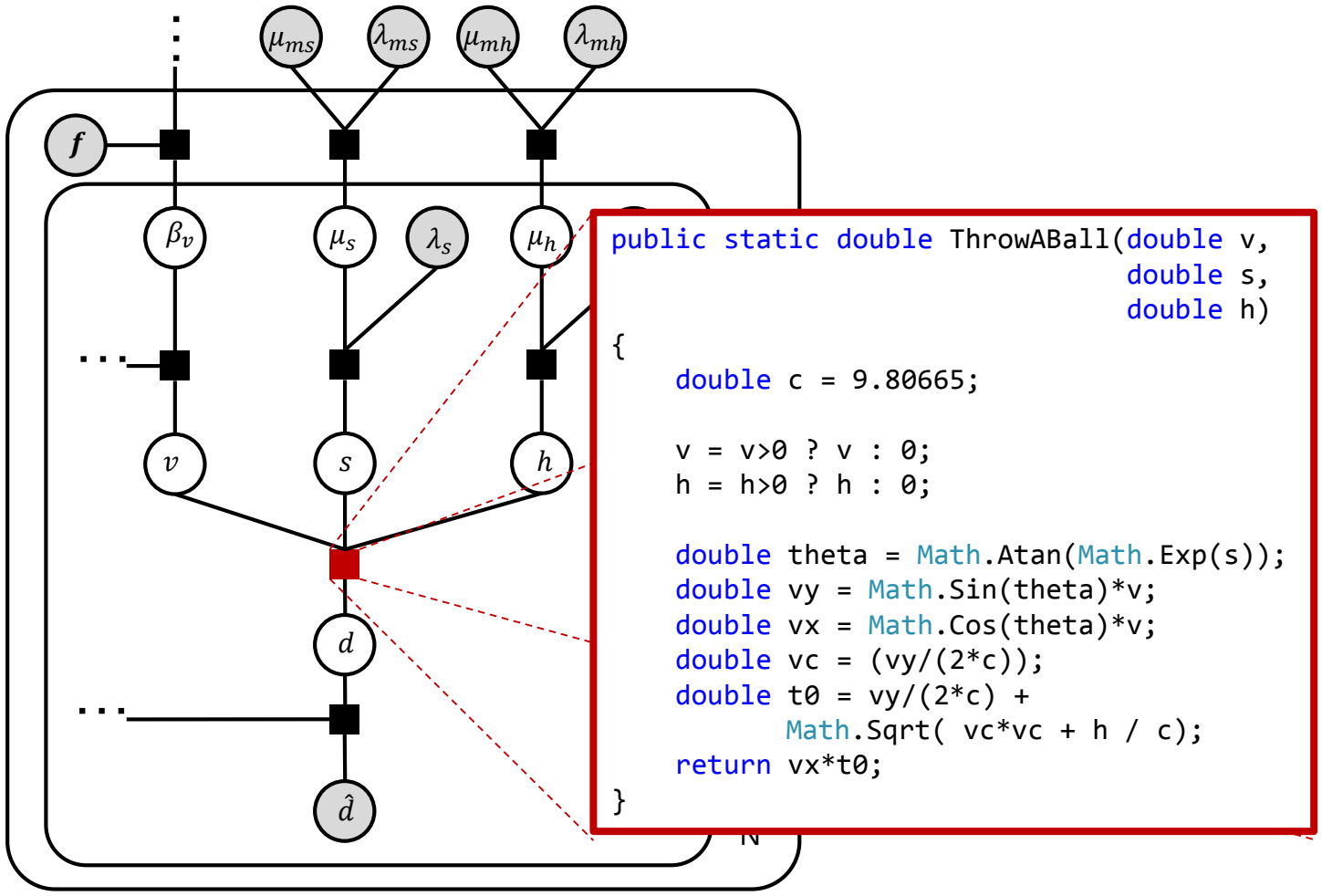




Speed up over sampling



Ball Throwing Simulation

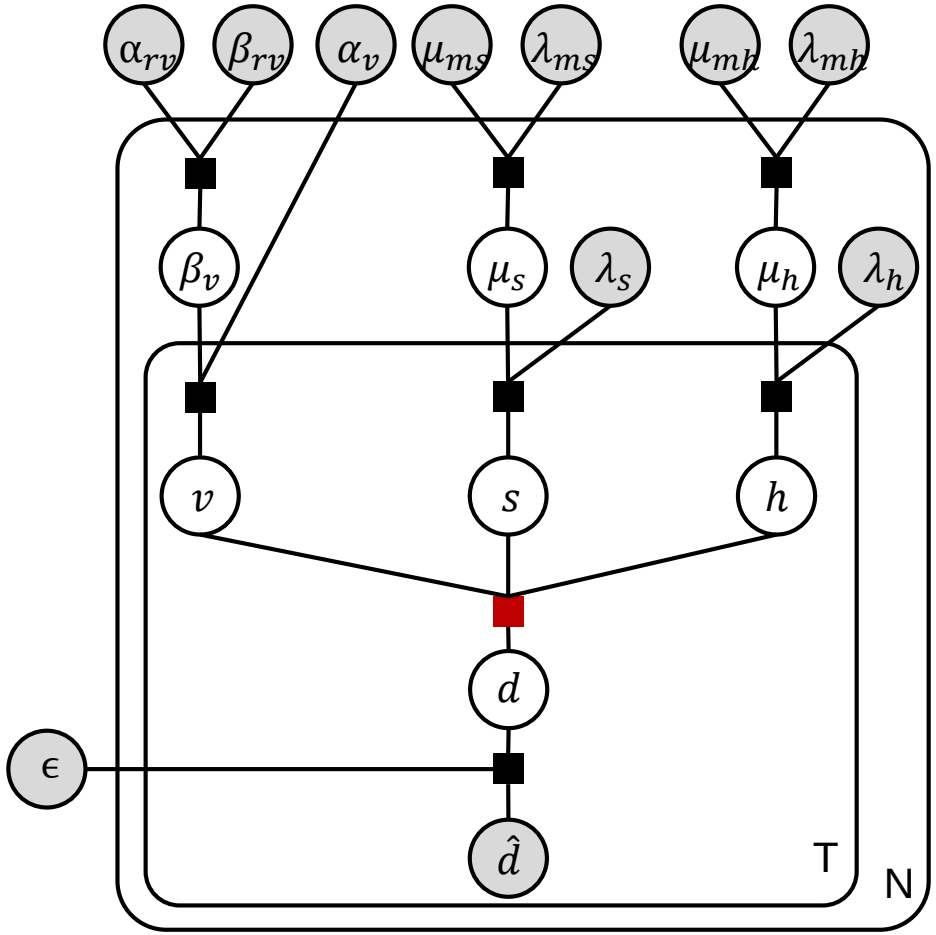


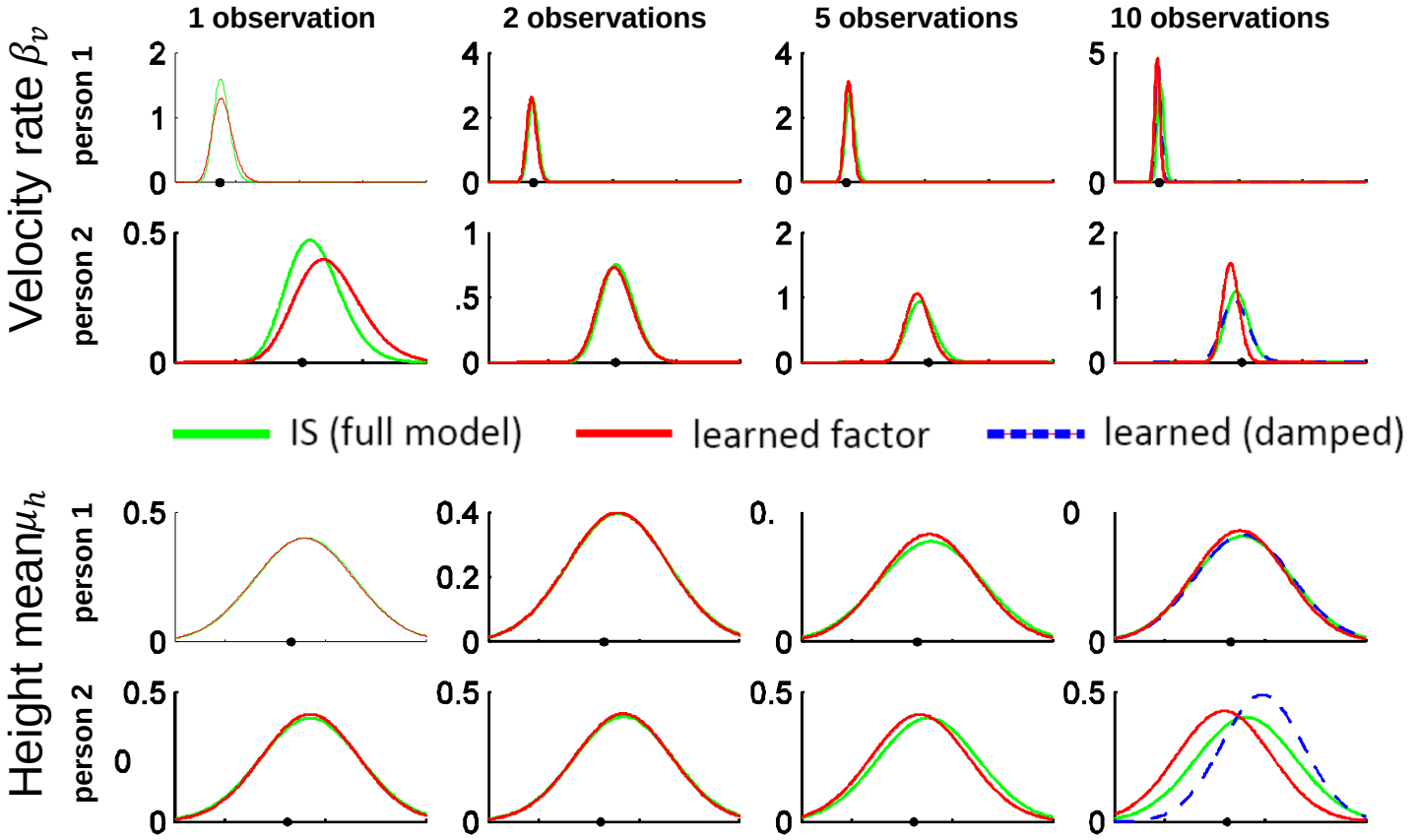

```
public static double ThrowABall(double v,
                                 double s,
                                 double h)
{
    double c = 9.80665;

    v = v>0 ? v : 0;
    h = h>0 ? h : 0;

    double theta = Math.Atan(Math.Exp(s));
    double vy = Math.Sin(theta)*v;
    double vx = Math.Cos(theta)*v;
    double vc = (vy/(2*c));
    double t0 = vy/(2*c) +
           Math.Sqrt( vc*vc + h / c);
    return vx*t0;
}
```

Ball Throwing Simulation





Conclusions

Probabilistic programs can…

- Solve complex problems,
- Be quick to write,
- Be fast to execute.

But more work is needed on…

- Speed
- Speed
- Speed.
- (& ease of use/reliability/debugging/training/examples…)

In the future, every programmer will be doing machine learning!

Thanks!

http://research.microsoft.com/infernet