"Fuzzy Controllers Compared with Linear Digital Controllers"

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1.0 Abstract

The purpose of any control system is to control the outputs of a plant in a manner described by the inputs to the control system. If a mathematical model for the plant is known, engineers can find the best linear controller to elicit the desired response. Linear controllers are well understood because they are described with solvable equations based on linear combinations of the inputs, outputs, and their derivatives. Other types of controllers are theoretically possible, but do not lend themselves to direct mathematical analysis. One such alternative to linear control is fuzzy control. Using fuzzy logic, a controller can be designed that outperforms linear controllers. The nonlinearities introduced, however, make it difficult to characterize the control system and therefore to determine relative stability or the optimality of the design.

2.0 Objective

The object of this paper is to describe how fuzzy logic controllers differ from linear controllers. It begins with an outline of linear control theory, and then compares the design of a linear digital controller with that of a fuzzy controller. It then discusses the problem of characterizing the response of a generally nonlinear control system. The conclusion assesses the strengths and weaknesses of either approach to control system design.

3.0 Theory

Every control system is designed to control its outputs in a manner prescribed by its inputs. There may be one or more inputs, and one or more outputs, but there is always at least one controller, one plant, and one feedback loop. There are many ways of configuring the control system, one of which is shown in Figure 1.

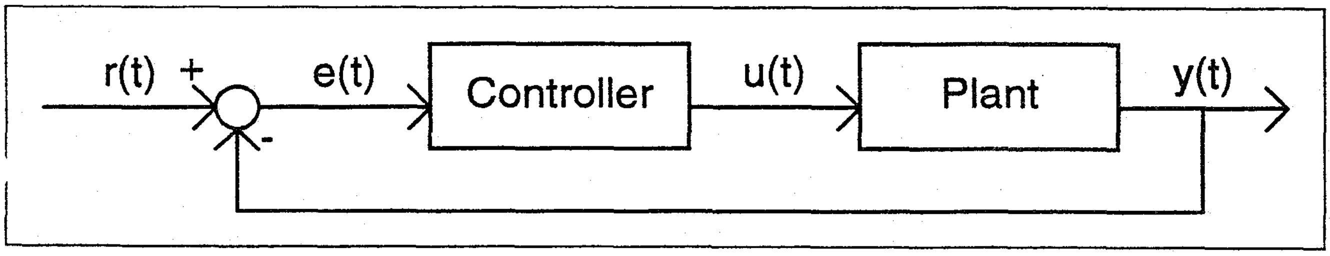

Figure 1: General Unity-Feedback Control System

The plant is the machine or device that is being controlled. A measurement device is assumed to be inside the plant that converts the object of control, such as a velocity or position, into an appropriate signal, such as a voltage potential.

The controller is the device to be designed. The controller acts upon the error signal, e(t), which is the difference between the input signal and the output signal. Ideally, in the steady state, the signal e(t) tends toward zero.

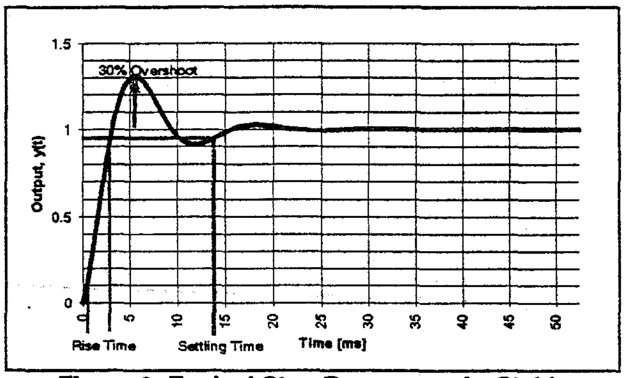

Figure 2: Typical Step Response of a Stable Control System

3.1 Control System Performance

If the control system is stable, a steady state value at the input will produce a steady state value at the output. A linear control system can be characterized by its step response, since superposition applies. The step response of a control system is a plot of its response to a sudden change in the input signal. A typical step response is shown in Figure 2. Important parts of the step response are the rise time, overshoot, and settling time. These quantities are defined in Equations (1) through (3).

$$t_{rise} = t_{90\%} - t_{10\%} \tag{1}$$

$$t_{settling} = t_{95\% < y(t) < 105\%} \tag{2}$$

$$overshoot = \frac{y_{max} - y_{ss}}{y_{ss}} \tag{3}$$

The rise time is a measure of how fast the control system's response is to a given input. The settling time is a measure of the relative stability of the system, and how long it takes the output to reach and stay within 5% of its final value. The overshoot is a measure of how much extra strain is put on the plant by the control system.

The control system whose step response is shown in Figure 2 has an overshoot of 30%, a rise time of 3 ms, and a settling time of 14 ms.

3.2 Linear Analog Controllers

The behavior of linear control systems is well understood. Designing a linear control system involves deriving a mathematical model for the plant and choosing an appropriate transfer function for the controller.

$$A_0 y(t) + A_1 \frac{d}{dt} y(t) + \cdots + A_n \frac{d^n}{dt^n} y(t) = B_0 x(t) + B_1 \frac{d}{dt} x(t) + \cdots + B_m \frac{d^m}{dt^m} x(t) \tag{5}$$

If the assumption is made that the controller can be described by an additional transfer function G(s), then the transfer function of the control system in Figure 1, given by Mason's formula, is given by

$$F(s) = \frac{G(s)H(s)}{1 + G(s)H(s)}. \tag{6}$$

Knowing this, the designer can choose an appropriate transfer function, G(s), to form a control system transfer function F(s) with desirable features, such as low overshoot and fast rise time.

3.3 Linear Digital Controllers

The discussion up to this point has been focused on analog controllers, but the same general methods used for analog controllers can be used for digital controllers. Figure 1 can be redrawn for a digital controller, as shown in Figure 3. The plant is still operating in continuous time, but the controller is acting in discrete time. It receives a stream of evenly spaced error signals, e_k, and produces a stream of evenly spaced control signals, u_k. The digital and analog portions are coupled with digital to analog and analog to digital converters, labeled D/A and A/D respectively.

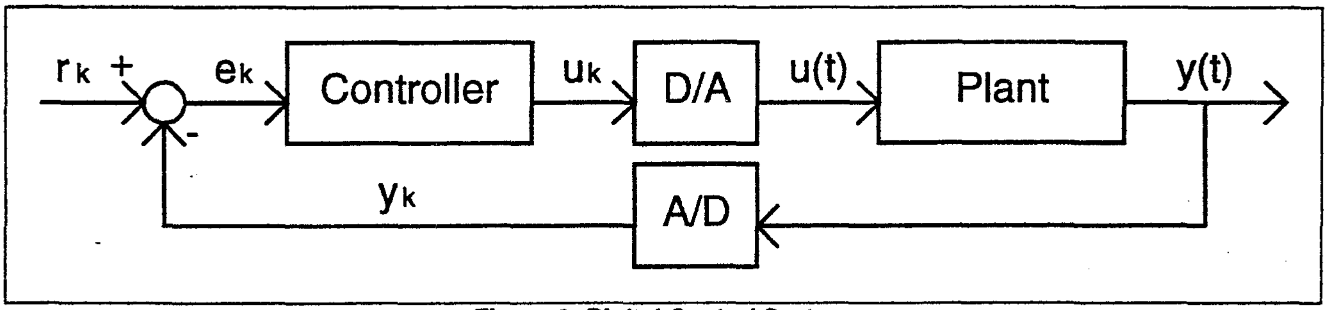

Figure 3: Digital Control System

The mathematical model of the plant can take the form of Equation (4), which describes the plant's frequency response.

$$H(s) = \frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)} \tag{4}$$

Transfer functions such as these are equivalent to Equation (5), wherein a linear combination of the output and its derivatives is equal to the input and a linear combination of its derivatives.

There are additional concerns when designing a digital controller, such as insuring that the sampling time is short enough. If care is taken on these critical matters, however, the control system in Figure 3 can be designed in much the same way as the continuous time control system.

3.4 Fuzzy Logic

Fuzzy logic allows the designer to specify the controller without knowing the exact model of the plant and without resorting to transfer functions.

A fuzzy logic controller can be divided into three parts, shown in Figure 4. The fuzzification and defuzzification blocks act like translators, translating exact signals into fuzzy signals and back again. The rule evaluation consists of transforming inputs to outputs in an intuitive way.

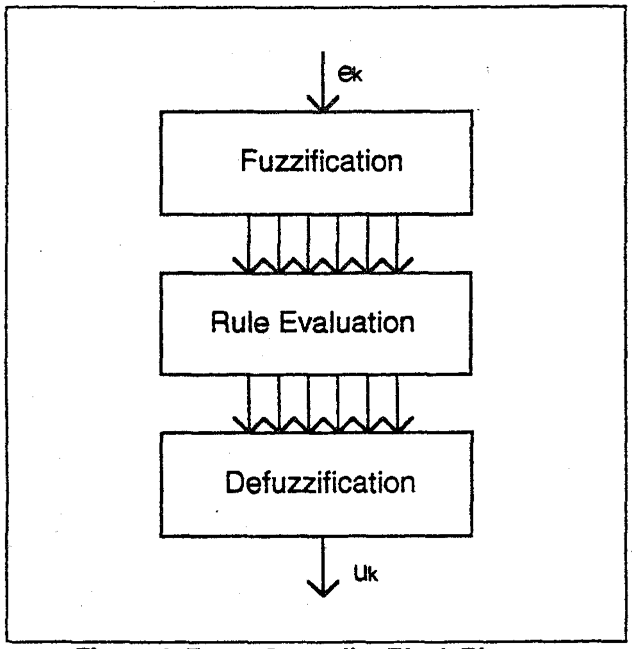

Figure 4: Fuzzy Controller Block Diagram

The inputs and outputs of the rule evaluation block are all fuzzy variables, each of which can take on any value from zero to one. A value close to zero would correspond to "mostly false," and a value close to one would correspond to "almost certainly true." A value near 0.5 would mean

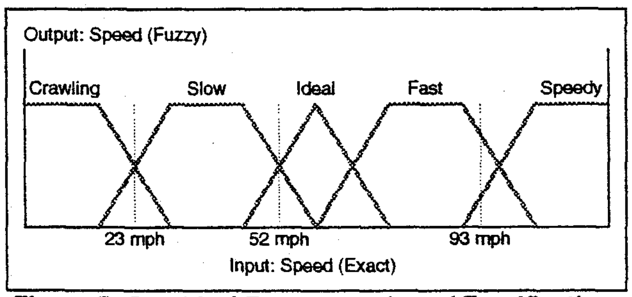

Figure 5: Graphical Representation of Fuzzification.

'somewhat true, and somewhat false." This process is straightforward. If, for example, there was a need to convert highway speed into fuzzy variables, Figure 5 could be used. If a car was traveling at 93 mph, the assertion, "The car's speed is fast," would be 0.75 true. The assertion, "The car's speed is speedy," would be 0.25 true. On the other hand, the assertions, "The car's speed is slow," or "The car's speed is ideal," would both be 0.0, totally false.

Using a set of functions like those of Figure 5, the fuzzification block creates a set of fuzzy variables that answer the questions, "How slow is the car," "How fast is the car," or "How speedy is the car?" These numbers are then used by the rule evaluation block.

The rule evaluation block tests the fuzzy inputs against a set of fuzzy rules, and produces a set of fuzzy output variables. A set of fuzzy rules describes action to be taken under different conditions. An example of a fuzzy rule would be, "If the car is crawling, push the accelerator very hard." It is not within the scope of this paper to analyze the combination of rules and fuzzy inputs to form the fuzzy outputs. The output variables all correspond to different states of the output of the controller, such as "acceleration should be increased" or "acceleration should be decreased." Each of these output variables, as fuzzy variables, has a degree of truth assigned to it.

The defuzzification block takes the signals from the fuzzy inference unit and combines them into a single value that is the output of the fuzzy controller. There are several ways of doing this, none of which is relevant.

In designing a controller this way, the engineer is freed from directly manipulating the frequency response of the control system. The output is determined not by a linear combination of inputs, but by evaluating inputs and rules in an intuitive, although highly nonlinear, way.

4.0 Characterization of Fuzzy Control Systems

Determining the response of a fuzzy control system is not as easy as doing the same for a linear control system. The equations of the controller do not resolve themselves into transfer functions, and are quite nonlinear. The best way to see how a fuzzy control system will behave is either to simulate or to implement it.

4.1 The Mathematics of a Fuzzy Controller

One way of visualizing a general, non-adaptive digital control system is as a function of one or more variables, usually the present and past values of the input signal, as in Equation (4).

$$u_k = f(e_k, e_{k-1}, e_{k-2}, \dots, e_{k-n}) \tag{4}$$

Given any sequence of inputs, there is a single control signal to be applied. If two such values are all that is necessary, one can imagine a control surface, where the height of the surface (dependent variable) is the control signal and the planar coordinates (independent variables) are the current and last inputs (see Figure 6).

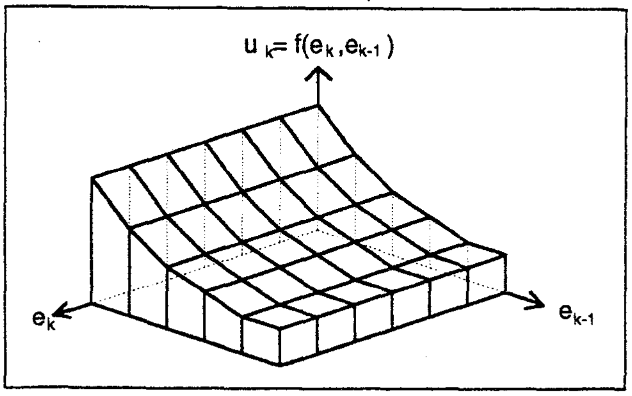

Figure 6: Control Surface With Two Independent Variables

If the function f is a linear combination of the independent variables, then a transfer function can be defined for the control system. Methods exist for optimizing this linear control surface to get minimum overshoot, minimum settling time, or other desired characteristics. These optimal values are, however, based on the restraint that the controller is linear. If a method exists for shaping the control surface in another way, better values may be obtained.

Fuzzy logic, in its simplest form, is nothing more than an intuitive tool for shaping the control surface.

4.2 Simulation of a Fuzzy Controller

If a fuzzy controller produced a linear control surface, it could be analyzed just like a linear controller. But this is not the case. Since the control surface is based on nonlinear equations that do not lend themselves to analysis, the only option left is simulation of the control system to observe its quality.

Simulation of a fuzzy control system involves a simulation of the fuzzy controller and a simulation of the plant.

Simulation of the fuzzy controller is not difficult. Both an initial set of membership functions and an initial rule base are chosen. Fuzzy inference algorithms are chosen that mimic the algorithms that will be used in the final control system.

Simulation of the plant involves developing a mathematical model of the plant. This model can be an approximate linearized model of the plant, or another fuzzy block that is known to specify the plant's behavior.

Typical control signals can be applied to the simulated system. Using the response of the control system as a guide, the fuzzy rules and membership functions can be manipulated to improve the system performance. The process can be repeated as necessary until a satisfactory response is achieved.

5.0 A Fuzzy Controller

To illustrate the ease of implementing a fuzzy controller, consider the design of a velocity controller for a high speed printer head. The author is currently involved in such a project.

The single input to the microprocessor is an interrupt that occurs with each 0.01 inch the print head travels. The microprocessor times the distance between interrupts, and uses this as an input to the fuzzy controller. The other input to the fuzzy controller is the change of this period from the previous period.

At this point, there is no mathematical model of the system. Theory predicts that the system will have at least two poles due to the nature of the DC motor. The location of these poles are unknown, but appropriate values for inputs and outputs are known. The input is normalized to a range of -128 to 127, to make the input membership functions easier to describe (see Figure 7).

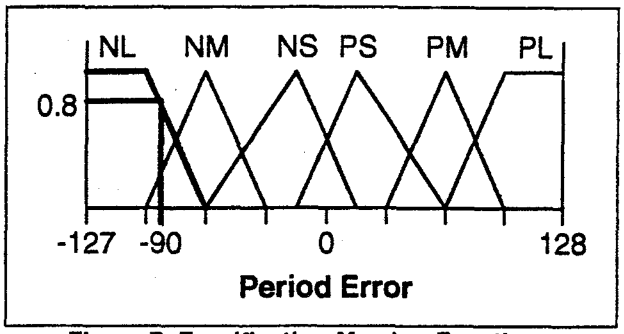

Figure 7: Fuzzification Member Functions

The output of the fuzzy controller will be an offset for the current control signal. The motor does not respond quickly, so member functions are chosen that are close to zero, Figure 8.

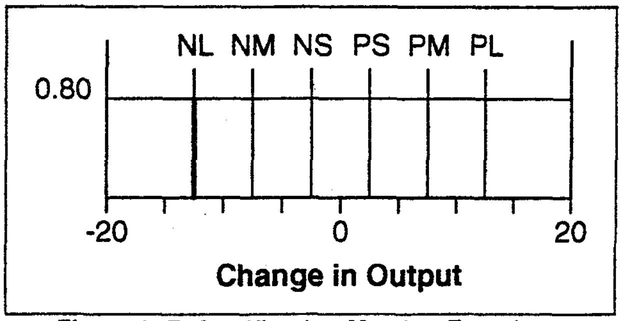

Figure 8: Defuzzification Member Functions

The next step in the fuzzy design is to choose appropriate rules, which seem to make sense. These rules are placed into a rule matrix that defines desirable outputs for various combinations of the inputs. A possible rule matrix is shown in Figure 9.

	NL	NM	NS	PS	PM	PL
NL	NL	NM	NM	NS	PS	PM
NS	NL	NM	NS	PS	PS	PM
PS	NM	NS	NS	PS	PM	PL
PL	NM	NS	PS	PM	PM	PL

TIME

DTIME

Figure 9: Rule Matrix for Fuzzy Controller

The rule matrix is easy to read. The rule in the first row and first column states: "If TIME is Negative Large and DTIME is Negative Large, then OUTPUT is Negative Large." Restated, "If the time it takes to travel 0.01 inch is way too small, and it is getting smaller, decrease the output a lot." In all, there are 24 rules in Figure 8, each of which is based only on assumptions of how the print head should behave.

A fuzzy controller similar to the one described above has been simulated and performs quite well. The system remained stable for a wide variety of pole placement in the controlled plant. When the poles of the plant were placed at 5 and 10 radians per second, the rise time was under 20 sample periods, and there was no overshoot. A comparable linear design was able to control the same plant with a rise time under 10 sample periods, with an overshoot of under 10%.

6.0 Conclusions

Linear control systems are well understood because they are based on solvable, linear ordinary differential equations. Given a transfer function approximation for the plant, an engineer can find an optimal transfer function for the controller. The advantage of this approach is that, if standard design procedures are followed, a stable and well-understood system is guaranteed. A major drawback to this approach is that the optimal transfer function is found, which may not be the optimal controller.

Fuzzy logic provides a way of shaping the control surface in a non-linear, non-arbitrary way. An acceptable controller is found by observing the performance of the control system with typical input functions and adjusting the fuzzy controller accordingly. The major advantage of using a fuzzy controller is that better control systems are possible than with the restriction of linearity. The disadvantage comes as a result of the nonlinearities introduced. The only practical way of predicting the response of the control system is through simulation.

7.0 References

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